Section 1.10

I. Electrical Networks.

V = Voltage (in volts V),
R = Resistance (in ohms Ω),
I = Current (in amperes amps)

1. **Ohm's Law:** $V = RI$

2. **Kirchhoff's Voltage Law:** "No voltage gets lost", i. e. the algebraic sum of voltage drops RI around a loop is equal to the algebraic sum of voltage sources around that loop (keeping the same direction).

3. **Direction of positive voltage:** from positive (longer) side of the battery to the negative (shorter) side.

4. **Careful:** A battery counts only if it is directly on the loop.

EXAMPLE: (Lay 1.10 # 7.) Determine the loop currents in the following network.

Solution:

$$\begin{array}{rrrrcr} 12I_1 & -7I_2 & & -4I_4 & = & 40 \\ -7I_1 & +15I_2 & -6I_3 & & = & 30 \\ & -6I_2 & +14I_3 & -5I_4 & = & 20 \\ -4I_1 & & -5I_3 & +13I_4 & = & -10 \end{array}$$

$$\begin{pmatrix} 12 & -7 & 0 & -4 & 40 \\ -7 & 15 & -6 & 0 & 30 \\ 0 & -6 & 14 & -5 & 20 \\ -4 & 0 & -5 & 13 & -10 \end{pmatrix} \implies \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{pmatrix} = \begin{pmatrix} 11.43 \\ 10.55 \\ 8.04 \\ 5.84 \end{pmatrix}$$

Kirchhoff's Current Law: Current in a branch is the algebraic sum of the loop currents in that branch.

EXAMPLE: In above network:

1. $I_1 - I_2 = 11.43 - 10.55 = 0.88$ amps
2. $I_3 - I_4 = 8.04 - 5.84 = 2.2$ amps
3. $I_2 - I_3 = 10.55 - 8.04 = 2.51$ amps.

II. Difference Equations.

Measurements are made on a system and the data is collected in a vector:

$$\begin{aligned} v_0 &= \text{initial state vector,} \\ v_1 &= \text{1st state vector,} \\ v_2 &= \text{2nd state vector,} \\ &\vdots \text{ etc} \end{aligned}$$

We have a **linear difference equation** (or **recurrence relation**) problem if there is a matrix A such that

$$\begin{aligned} v_1 &= A\,v_0, \\ v_2 &= A\,v_1, \\ v_3 &= A\,v_2, \\ &\vdots \text{ etc} \end{aligned}$$

EXAMPLE: (Lay 1.10 # 11.)

At the beginning of 1990, the population of California was $29,716,000$, and the population livinig in the US but outside California was $218,994,000$. During the year, $509,500$ persons moved from California to elsewhere in the US, while $564,100$ persons moved to California from elsewhere in the US.

a) Set up the migration matrix for this situation using 5 decimal places for the rates.

b) Compute the projected populations in the year 2000 for California and elsewhere in the US, assuming that the rates remained constant during the 10-year period.

Solution:

a)

$$\frac{509,500}{29,716,000} \approx 0.01715 \text{ (California to Outside)}$$

$$1 - 0.01715 = 0.98285 \text{ (California to California)}$$

$$\frac{564,100}{218,994,000} \approx 0.00258 \text{ (Outside to California)}$$

$$1 - 0.00258 = 0.99742 \text{ (Outside to Outside)}$$

Thus:

$$\begin{array}{cc} & \begin{array}{ccc}\text{From:} & \text{CA} & \text{Out}\end{array} \\ A = & \begin{pmatrix} .98285 & .00258 \\ .01715 & .99742 \end{pmatrix} \quad \begin{array}{c}\text{To:} \\ \text{CA} \\ \text{Out}\end{array}\end{array}$$

b)

Initial state vector in 1990: $x_0 = \begin{pmatrix} 29.716 \\ 218.994 \end{pmatrix} \begin{array}{c}\text{CA} \\ \text{Out}\end{array}$ in millions of persons.

In 1991: $x_1 = A\,x_0 = \begin{pmatrix} .98285 \cdot 29.716 + .00258 \cdot 218.994 \\ .01715 \cdot 29.716 + .99742 \cdot 218.994 \end{pmatrix} \approx \begin{pmatrix} 29.7\ldots \\ 219.0\ldots \end{pmatrix}$

In 1992: $x_2 = A\,x_1 \approx \ldots$

$\vdots$

In 2000: $x_{10} = A\,x_9 \approx \begin{pmatrix} 30.223\ldots \\ 218.487\ldots \end{pmatrix}$ in millions.