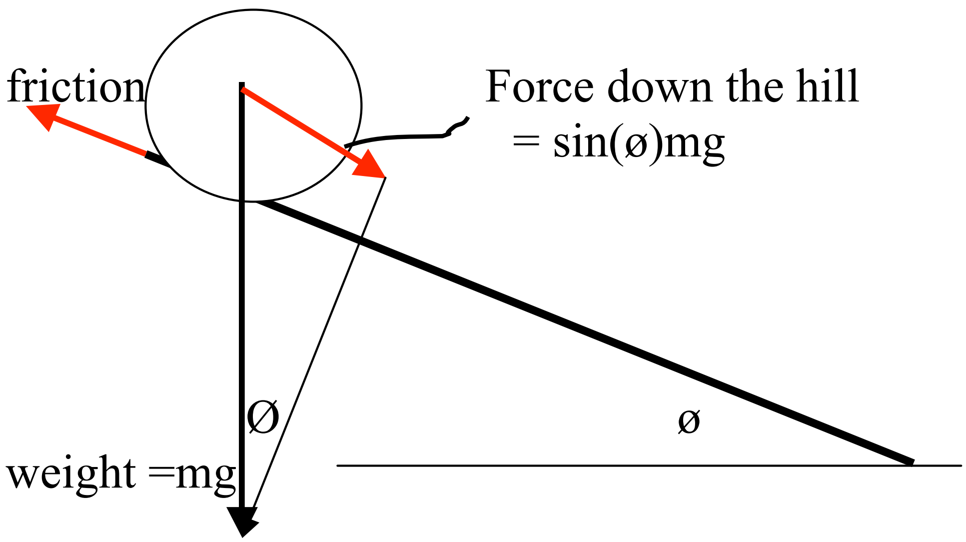
Further thought-Moment of inertia race

Starting from rest in a race down a hill, which will win first, the hoop or the disc? No matter what the mass or the radius of the hoop or disc, the disc will always win.

The moment of inertia I of a hoop is defined as:

$$I_{hoop} \equiv mR^2$$

Where m is the mass of the hoop and R is the radius of the hoop.



The friction up the incline will apply a torque to the hoop that will give it an angular acceleration down the hill.

$$\text{Torque} \equiv F_{\perp} d$$

In this example $F_{\perp}$ is the friction up the hill and d is the radius R of the hoop.

$\text{Torque} = f \cdot R$ and since $\text{Torque} = I\alpha$, $f \cdot R = I \cdot \alpha$

Solving for friction:

$$f = \frac{I\alpha}{R}$$

From Newton's second law we know that $\sum F = ma$

The component of the weight down plus the friction up the hill is given by:

$$mgSin\theta + -f = ma$$

Next we substitute for the friction f.

$$mgSin\theta + -\frac{I\alpha}{R} = ma$$

Remember that α is the angular acceleration of the hoop and is related to the linear acceleration by:

$$\alpha = \frac{a}{R}$$

Now substitute for the angular acceleration

$$mgSin\theta + -\frac{Ia}{R^2} = ma$$

Since $I_{hoop} \equiv MR^2$

WE substitute for the moment of inertia of the hoop.

$$mgSin\theta + -\frac{mR^2a}{R^2} = ma$$

Dividing out the mass and the radius of the hoop we now have:

$$gSin(\o) - a = a$$

$$a_{hoop} = \frac{gSin\theta}{2}$$

We find that the acceleration of the hoop down the hill does not depend on its mass or its radius. In this case, mass does not matter.

Doing similar calculations we can show that the acceleration of the disk will be:

$$a_{disk} = \frac{2}{3} g \mathrm{Sin}(ø)$$

In conclusion, the acceleration of the disk will always be larger than that of the hoop-no matter what their masses or radii are!!

Professor Matt Enjalran helped me with this solution.

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